Robust Stability of Teleoperation Schemes Subject to Communication Delays

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Abstract

This paper addresses the robust stability analysis of some bilateral teleoperation control scheme subject to various constant and/or time-varying delays in the communication channel.
The stability conditions are derived using *frequency-domain* techniques. More specifically, in the case of constant delays, the stability regions of the systems' parameters are completely characterized.
Finally, the analysis is extended to the case of time-varying uncertain delay, and we derive sufficient (closed-loop) stability conditions.

1 Introduction

A basic *teleoperation system* consists of a *slave device* and a *master device*. The master is *directly manipulated* by a *human operator*, and the slave is designed to track the master closely. The main purpose of such a master-slave configuration is to manipulate the environment (or space) generally inaccessible to human operators, such as hazardous environment. Such systems are often known as a *bilateral teleoperator* systems.

Time delay plays an important role in the teleoperation systems. Due to the physical distance between the master and slave, as well as the signal processing, the communications involve significant delays. Another source of delay is the reaction of the human operators. In this chapter, we will discuss the effect of the communication delays (constant or time-varying) on the closed-loop stability of such systems.

In this context, we are interested in characterizing the way that delays change performances in communication channels connecting the master and slave sites (*bilateral teleoperation*). It is well known that the *passivity* of the channel (see, e.g., [1, 6, 17, 18, 24]) may be used to guarantee desirable characteristics for the closed-loop schemes (see also [10]). The techniques proposed to perform such an analysis use the scattering transformation [1] or the wave variable transformation [17, 18], if the delays are assumed constant. The case of time-varying or distributed delays was considered in [11, 19] using the wave transformation approach and in [14] but under some assumptions on the delay variation.

Consider the following equations widely used to describe the dynamics of teleoperators [1, 11].

$$\begin{cases} M_m\ddot{x}_m(t) + B_m\dot{x}_m(t) = F_h(t) - F_m(t) \\ M_s\ddot{x}_s(t) + B_s\dot{x}_s(t) = F_s(t) - (1+\alpha_f)Z_e\dot{x}_s(t), \end{cases} \tag{1}$$

where $\dot{x}, M, B$ are the velocities, inertias, and damping coefficients, respectively. The subscripts m and s denote the corresponding quantity is of the master and the slave, respectively. The input F_h denotes the operator force or torque, and Z_e is the environmental impedance. The quantity F_s is the force or torque applied to the slave transmitted from the master, and F_m is the force on the master fed back from the slave.

For an explicit stability analysis, see [6] for various frequency-domain techniques (see also [12]), and [2] for a Lyapunov functional approach. For delay-independent stability, the approach proposed in this paper is simpler than the one proposed in [6], and the derived conditions are *necessary and sufficient*, and in an *analytical* form.

For delay-independent stability, the main idea is to use a frequency-domain method based on the Tsypkin's criterion [5, 12]. For frequency-sweeping tests applied to various control systems, see, for instance, [3]. Various discussions and comments related to such techniques can be found in [12]. Such an approach was used in [20] for the closed-loop stability analysis of a simple teleoperation control scheme, where delay-independent/delay-dependent stability conditions were derived under the assumption of symmetric delays in the channels ($\tau_1 = \tau_2 = \tau$). The approach considered here follows the lines of the approach mentioned above, but it includes also the extension to the case when the delay is assumed to be time-varying.

As in [11], consider the control law described by the follo-

*work in collaobration with KEQIN GU, Southern Illinois University at Edwardsville, USA, and DAMIA TAOUTAOU, Université de Technologie de Compiègne wing equations

$$F_s(t) = K_s \int_0^t (\dot{x}_{sd}(t) - \dot{x}_s(t))d\theta + B_{s2}(\dot{x}_{sd}(t) - \dot{x}_s(t)), \quad (2)$$

$$F_m(t) = K_m \int_0^t (\dot{x}_m(t) - \dot{x}_{md}(t))d\theta + B_{m2}(\dot{x}_m(t) - \dot{x}_{md}(t)). \quad (3)$$

Due to communication delays, the most recently available information is used instead, that is, we choose,

$$\dot{x}_{sd}(t) = \dot{x}_m(t - \tau_1), \quad (4)$$
$$\dot{x}_{md}(t) = \dot{x}_s(t - \tau_2), \quad (5)$$

where τ_1 and τ_2 are the delays in the forward and feedback communication channels, respectively.

As mentioned above, we are interested in first finding *analytical conditions* on the system's parameters such that the closed-loop system is asymptotically stable for arbitrary communication delays. For those parameters which do not satisfy such delay-independent stability conditions, we will find the corresponding *delay intervals* such that the closed-loop system is stable. Furthermore, we are also interested in finding conditions for which there is only one delay interval, and computing the corresponding *optimal bounds*. A similar problem, but only with constant and symmetric time-delays (τ_1, τ_2), was considered in [20].

In the case of time-varying delay uncertainty, the idea is to construct an appropriate fictitious transfer function such that the stability of the original closed-loop scheme is reduced to some H_∞-norm property of the corresponding transfer. To the best of the authors' knowledge, such an approach was not considered in the bilateral teleoperation case.

The paper is organized as follows: Section 2 is devoted to the stability analysis of the closed-loop system using frequency-domain techniques. Constraints on the controller's gain K_s and 'damping' B_{s1} will be given such that the closed-loop scheme is asymptotically stable *independent* of the *communication delays*. Next, the *delay-dependent* stability of the closed-loop system will be considered. Section 3 discusses the case of time-varying delays. Some concluding remarks end the paper. The notations are standard.

2 Stability Analysis for Constant Delays

2.1 Problem setup

Carrying out the Laplace transform (under zero initial conditions) of the closed-loop system, using the velocities $v_m(t) = \dot{x}_m(t)$ and $v_s(t) = \dot{x}_s(t)$ as the system variables, we obtain

$$\begin{cases} M_m s V_m(s) + B_m V_m(s) = F_h(s) - F_m(s), \\ M_s s V_s(s) + B_s V_s(s) = F_s(s) - (1+\alpha_f) Z_e V_s(s), \end{cases} \quad (6)$$

and

$$F_s(s) = \frac{K_s + B_{s2}s}{s} \mathbf{e}^{-\tau_1 s} V_m(s) - \frac{K_s + B_{s2}s}{s} V_s(s). \quad (7)$$

$$F_m(s) = \frac{K_m + B_{m2}s}{s} V_m(s) - \frac{K_m + B_{m2}s}{s} \mathbf{e}^{-\tau_2 s} V_s(s). \quad (8)$$

Using the control laws (7) and (8) in the second equation of (6), with the notation $\overline{B}_s = B_s + (1+\alpha_f)Z_e$, it follows that:

$$V_s(s) = \frac{K_s + B_{s2}s}{M_s s^2 + (\overline{B}_s + B_{s2})s + K_s} \mathbf{e}^{-\tau_1 s} V_m(s), \quad (9)$$

Let $\tau = \tau_1 + \tau_2$, and use the following notations:

$$\Gamma_1(s) = B_{s2}s + K_s : \text{slave torque}, \quad (10)$$
$$\Gamma_2(s) = M_s s + \overline{B}_s : \text{slave}, \quad (11)$$
$$\Gamma_3(s) = M_m s + B_m : \text{master}, \quad (12)$$
$$\Gamma_4(s) = B_{m2}s + K_m : \text{master torque}, \quad (13)$$

we obtain from the first equation of (6) and (8)

$$V_m(s)\Gamma_3(s) = F_h(s) + \frac{\Gamma_4(s)}{s}\left(\mathbf{e}^{-\tau_2 s} V_s(s) - V_m(s)\right).$$

Using (9) in the above, we obtain

$$V_m(s) \cdot \left(\frac{s\Gamma_3(s) + \Gamma_4(s)}{s} - \mathbf{e}^{-\tau s} \frac{\Gamma_4(s)\Gamma_1(s)}{s(\Gamma_1(s) + s\Gamma_2(s))} \right) = F_h(s) \quad (14)$$

Therefore, the transfer function from F_h to V_m is given by:

$$H_1(s) = \frac{1}{\frac{(s\Gamma_3(s)+\Gamma_4(s))}{s}\left(1 - \frac{\mathbf{e}^{-\tau s}\Gamma_4(s)\Gamma_1(s)}{\Gamma_1(s)+s\Gamma_2(s)} \frac{1}{(s\Gamma_3(s)+\Gamma_4(s))}\right)}. \quad (15)$$

Furthermore, based on the form of $V_s(s)$, the transfer function from F_h to V_s is given by:

$$H_2(s) = H_1(s) \cdot \frac{K_s + B_{s1}s}{M_s s^2 + (\overline{B}_s + B_{s1})s + K_s} \mathbf{e}^{-\tau_1 s}. \quad (16)$$

Since $M_s, B_s, B_{s1}, K_s, \alpha_f, Z_e$ are positive real numbers, $H_1(s)$ and $H_2(s)$ share the right half plane poles. Therefore, to study the stability of the closed-loop system, it is sufficient to study the stability of the transfer function $H_1(s)$. Or, equivalent, one needs only to study the distribution of zeros of the expression:

$$1 - \mathbf{e}^{-\tau s} \frac{\Gamma_4(s)\Gamma_1(s)}{\Gamma_1(s) + s\Gamma_2(s)} \frac{1}{(s\Gamma_3(s) + \Gamma_4(s))}. \quad (17)$$

We will first study asymptotic stability of the closed-loop system when it is *free from delays*. In this case, the zeros of the characteristic function (17) becomes those of the third-order polynomial:

$$P(s) = s\Gamma_2(s)\Gamma_3(s) + \Gamma_3(s)\Gamma_1(s) + \Gamma_4(s)\Gamma_2(s). \quad (18)$$

Using the Routh-Hurwitz stability criterion (see, for example, [8]), it follows that the system free from delays is asymptotically stable if and only if the following inequality holds:

$$\begin{aligned}&\left(\frac{K_m}{M_m}+\frac{K_s}{M_s}+\frac{B_mB_{s2}}{M_mM_s}+\frac{\overline{B}_s(B_{m2}+B_m)}{M_sM_m}\right)\\&\cdot\left(\frac{B_{s2}+\overline{B}_s}{M_s}+\frac{B_{m2}+B_m}{M_m}\right)\\&>\frac{K_sB_m+K_m\overline{B}_s}{M_sM_m}.\end{aligned} \tag{19}$$

It is not difficult to show that (19) is always valid for all positive parameters. Therefore, as expected, if the system is free from delay, the controller (2)-(5) guarantees the *asymptotic stability* of the closed-loop system.

2.2 Delay-independent stability

The next step is to find the conditions under which the stability in the closed-loop systems is guaranteed for *arbitrary communication delays* τ_1 and τ_2. First, under a certain parameter constraint, we will find necessary and sufficient conditions for stability. Next, we will provide a *simple* sufficient condition easy to use in practice.

Theorem 1 *Assume the feedback gains K_m and K_s, B_{m2} and B_{s2} are positive constants. Then the closed-loop system is asymptotically stable for all communication delays $\tau_1,\ \tau_2$ if and only if, $\forall\omega>0$:*

$$\mid (j\omega)\Gamma_3(j\omega)+\Gamma_4(j\omega)\mid > \left|\frac{\Gamma_4(j\omega)\Gamma_1(j\omega)}{\Gamma_1(j\omega)+j\omega\Gamma_2(j\omega)}\right|. \tag{20}$$

Proof: In view of the form of $H_1(s)$, since $s\Gamma_3(s)+\Gamma_4(s)$ is Hurwitz stable, it follows that the stability of the closed-loop system (1)-(5) is equivalent to the stability of the unit feedback closed-loop system with the open-loop transfer function

$$H_o(s)=\frac{\Gamma_4(s)\Gamma_1(s)}{(s\Gamma_3(s)+\Gamma_4(s))(\Gamma_1(s)+s\Gamma_2(s))}e^{-s\tau}. \tag{21}$$

Since $(s\Gamma_3(s)+\Gamma_4(s))(\Gamma_1(s)+s\Gamma_2(s))$ is Hurwitz stable, and $H_o(s)$ is strictly proper for $\tau=0$, then we may apply the Tsypkin's criterion, and the condition (20) follows directly.

Note that for $\omega=0$,

$$|j\omega\Gamma_3(j\omega)+\Gamma_4(j\omega)|=\left|\frac{\Gamma_4(j\omega)\Gamma_1(j\omega)}{\Gamma_1(j\omega)+j\omega\Gamma_2(j\omega)}\right|=K_m$$

Furthermore, if (20) is verified for $\omega>0$, then the same inequality holds for $\omega<0$. The condition (20) in Theorem 1 is a simple *frequency-sweeping test* that can be easily performed if the parameters of the system and the controller are given. To obtain a even simpler criterion than (20), introduce the notation

$$\gamma(K_m,B_{m2},K_s,B_{s2})=\sup_{\omega>0}\left|\frac{\Gamma_4(j\omega)\Gamma_1(j\omega)}{\Gamma_1(j\omega)+j\omega\Gamma_2(j\omega)}\right|, \tag{22}$$

which depends continuously on the controller's parameters K_m,B_{m2},K_s,B_{s2} (they are all real and positive). Then, we have the following natural corollary:

Corollary 1 *The closed-loop system is asymptotically stable for arbitrary communication delays $\tau_1,\tau_2\geq 0$ if the controller gains K_s,K_m and the "damping coefficients" B_{s2},B_{m2} are chosen to satisfy*

$$K_m<\frac{(B_m+B_{m2})^2}{2M_m} \tag{23}$$

$$\gamma(K_s,B_{s2},K_m,B_{m2})\leq K_m \tag{24}$$

Proof: The result is a straighforward from Theorem 1: The condition (23) ensures that $\mid j\omega\Gamma_3(j\omega)+\Gamma_4(j\omega)\mid$ is a strictly increasing function of ω, which implies $K_m<\mid j\omega\Gamma_3(j\omega)+\Gamma_4(j\omega)\mid$ for all $\omega>0$. Therefore the condition (20) is implied by (24).

As given in the next Proposition, the condition (24) can be written out explicitly.

Proposition 1 *The closed-loop system is asymptotically stable for all communication delays $\tau_1,\tau_2\geq 0$, if the controller's parameters satisfy:*

$$K_s\leq\frac{M_sK_m^2}{B_{m2}^2}\left(\sqrt{1+\frac{B_{s2}^2B_{m2}^2}{M_s^2K_m^2}\left(\left(1+\frac{\overline{B}}{B_{s2}}\right)^2-1\right)}-1\right) \tag{25}$$

$$K_m\geq\frac{B_{s2}B_m}{M_s} \tag{26}$$

$$K_m<\frac{(B_m+B_{m2})^2}{2M_m} \tag{27}$$

Proof: We will show that (25) and (26) is necessary and sufficient condition for (24), which will be sufficient to complete the proof. Define

$$\begin{aligned}&f:[0,\infty)\mapsto(0,\infty)\\&f(\omega^2)=\frac{\mid\Gamma_4(j\omega)\mid^2\cdot\mid\Gamma_1(j\omega)\mid^2}{\mid\Gamma_1(j\omega)+j\omega\Gamma_2(j\omega)\mid^2}.\end{aligned} \tag{28}$$

Then, $f(\omega^2)$ is in the form of

$$f(\omega^2)=K_m^2\frac{a\omega^4+b\omega^2+1}{d\omega^4+e\omega^2+1}$$

where the denominator

$$d\omega^4+e\omega^2+1>0 \text{ for all } \omega^2>0 \tag{29}$$

Therefore, the equation (23), or equivalently, $f(\omega^2)\leq K_m^2$, is equivalent to

$$a\omega^4+b\omega^2+1\leq d\omega^4+e\omega^2+1 \text{ for all } \omega^2>0$$

in view of (29). But the above is satisfied if and only if

$$a\leq d \tag{30}$$

$$b\leq e \tag{31}$$

With the specific parameters substituted, (30) reduces to (26). The condition (31) is a quadratic inequality of K_s, which is satisfied if and only if (25) is satisfied in view of the fact that K_s is positive.

Remark 1 (Tuning parameters) *Proposition 1 above gives a very simple way of constructing the controller (1) such that the closed-loop system is guaranteed to be asymptotically stable for all communication delays* $\tau_1,\ \tau_2 \geq 0$.

2.3 Delay-dependent stability

If (20) is not satisfied for all $\omega > 0$, the conditions for Theorem 1 do not hold, and there must exist delays such that the system is unstable. Since the system without delays is asymptotically stable, there always exists one or more intervals of delay such that the system is asymptotically stable. We are interested in finding the maximum $\tau^* > 0$ such that the system is asymptotically stable for all $\tau \in [0, \tau^*)$. This can be carried out by solving the equation

$$|\ (j\omega\Gamma_3(j\omega) + \Gamma_4(j\omega))\ |^2 = \left|\frac{\Gamma_4(j\omega)\Gamma_1(j\omega)}{\Gamma_1(j\omega) + j\omega\Gamma_2(j\omega)}\right|^2 \tag{32}$$

This equation can be reduced to a third order polynomial equation of the variable ω^2, and formulas are available to express the solutions explicitly (see, for example, [22]). Clearly, since (20) is not satisfied for all $\omega \geq 0$, and it is clearly satisfied for sufficiently large ω, the equation (32) has at least one real positive solution. Let all the real positive solutions be denoted as ω_i, $i = 1, 2, ..., m$. Clearly, $1 \leq m \leq 3$. Then, we can conclude:

Theorem 2 (Switch characterizations) *If (20) is not satisfied for all* $\omega > 0$*, let*

$$\tau^* = \min_{\ell \in \mathbf{Z}} \min_{1 \leq i \leq m} \frac{1}{\omega_i}\left[Log\left(\frac{\Gamma_4(j\omega)\Gamma_1(j\omega)}{(j\omega\Gamma_3(j\omega) + \Gamma_4(j\omega))(j\omega\Gamma_2(j\omega) + \Gamma_1(j\omega))}\right) + 2\pi\ell\right] > 0, \tag{33}$$

where "Log" denotes the principal value of the logarithm. Then, the closed-loop system is asymptotically stable for all $\tau \in [0, \tau^*)$.

Proof: As discussed above, the equation (32) has one to three real positive solutions. If and only if ω is a real positive solution, there exists a τ satisfying the characteristic equation

$$(s\Gamma_3(s) + \Gamma_4(s)) - \mathbf{e}^{-\tau s}\frac{\Gamma_4(s)\Gamma_1(s)}{\Gamma_1(s) + s\Gamma_2(s)} = 0$$

for $s = j\omega$, some simple but tedious computations lead to the smallest $\tau > 0$ in (33). Specific discussions on deciding the stable delay intervals are very similar to [16].

3 Time-Varying Uncertain Delays

Introduce the vector of state variables $x = [x_1, ..., x_4]^T$, where

$$x_1(t) = \int_0^t v_m(\theta)d\theta,\ \ x_2(t) = v_m(t) \tag{35}$$

$$x_3(t) = \int_0^t v_s(\theta)d\theta,\ \ x_4(t) = v_s(t) \tag{36}$$

Then, the closed-loop system described by (1) to (5) can be written as

$$\dot{x}(t) = Ax(t) + B_1x(t-\tau_1) + B_2x(t-\tau_2) + B_3F_h(t) \tag{37}$$

where:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{K_m}{M_m} & -\frac{B_m+B_{m2}}{M_m} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\frac{K_s}{M_s} & -\frac{(Bs+B_{s2}+(1+\alpha_f)Z_e)}{M_s} \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{K_s}{M_s} & \frac{B_{s2}}{M_s} & 0 & 0 \end{bmatrix},\ B_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{K_m}{M_m} & \frac{B_{m2}}{M_m} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{39}$$

and $B_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}^T$.

In the sequel, we will consider the case that the time-delays τ_1 and τ_2 are subject to time-varying uncertainties. Let $\delta_1(t)$ and $\delta_2(t)$ be continuous time-varying bounded functions with bounded derivatives,

$$0 \leq \delta_i(t) \leq \epsilon_i,\ \ \dot{\delta}_i(t) \leq \rho_i, 0 \leq \rho_i < 1\ \ i = 1, 2. \tag{40}$$

With the delay uncertainty, we write the system as follows:

$$\dot{x}(t) = Ax(t) + B_1x(t - \tau_1 - \delta_1(t)) + B_2x(t - \tau_2 - \delta_2(t)) \tag{41}$$

We have also omitted the human input term F_h since it does not affect the stability analysis in the state-space form. Although not considered here, it is also possible to allow δ_i to assume both positive and negative values with potential further reduction of conservatism, see [7]. Equation (41) can be written as:

$$\begin{aligned} \dot{x}(t) = {} & Ax + B_1x(t-\tau_1) + B_2x(t-\tau_2) \\ & - B_1\int_{-\delta_1(t)}^0 \frac{\partial}{\partial\theta}x(t-\tau_1+\theta)d\theta \\ & - B_2\int_{-\delta_2(t)}^0 \frac{\partial}{\partial\theta}x(t-\tau_2+\theta)d\theta \end{aligned} \tag{42}$$

Use (41) for the terms $\frac{\partial}{\partial\theta}x(t-\tau_1+\theta)$ and $\frac{\partial}{\partial\theta}x(t-\tau_2+\theta)$ in the above equation, (known as the model transformation) and let

$$u_1(t) = A\int_{-\delta_1(t)}^0 x(t-\tau_1+\theta)d\theta \tag{43}$$

$$u_3(t) = A\int_{-\delta_2(t)}^0 x(t-\tau_2+\theta)d\theta \tag{44}$$

$$u_2(t) = B_2 \int_{-\delta_1(t)}^{0} x(t-\tau_1+\theta-\tau_2-\delta_2(t-\tau_1+\theta))d\theta \tag{45}$$

$$u_4(t) = B_1 \int_{-\delta_2(t)}^{0} x(t-\tau_2+\theta-\tau_1-\delta_1(t-\tau_2+\theta))d\theta \tag{46}$$

Since $B_1B_1 = B_2B_2 = 0$, we can write (41) as :

$$\begin{aligned}\dot{x}(t) = Ax + B_1x(t-\tau_1) + B_2x(t-\tau_2) - B_1u_1(t) \\ -B_1u_2(t) - B_2u_3(t) - B_2u_4(t)\end{aligned} \tag{47}$$

Assuming zero initial conditions, we will estimate the gains from x to u_i, $i = 1,2,3,4$. It is useful to define $\nu_i(\eta) = \eta - \delta_i(\eta)$, $i = 1,2$. Then,

$$\eta - \varepsilon_i \le \nu_i(\eta) \le \eta$$

Also, since $d\nu_i/d\eta = 1 - \delta_i'(\eta) \ge 1 - \rho_i > 0$, ν_i is a strictly increasing function, the inverse function $\eta = \eta(\nu_i)$ is well defined, and

$$\frac{\partial \eta}{\partial \nu_i} = \frac{1}{1-\delta_i'(\eta)} \le \frac{1}{1-\rho_i}$$

Furthermore, due to the range of δ_i, we can easily verify that

$$\nu_i \le \eta(\nu_i) \le \nu_i + \epsilon_i$$

Using Jensen's Inequality [23] [7], we can show that:

$$\begin{aligned}&\int_0^t u_4^T(\xi)u_4(\xi)d\xi \\ &\le \int_0^t \delta_2(\xi)[\int_{-\delta_2(\xi)}^{0} (x^T(\nu_1(\xi-\tau_2+\theta)-\tau_1)B_1^T \cdot \\ &\qquad B_1x^T(\nu_1(\xi-\tau_2+\theta)-\tau_1))d\theta]d\xi\end{aligned} \tag{51}$$

Change integration variable from θ to μ, with $\mu = v_1(\xi - \tau_2 + \theta) - \tau_1$. Then, we have

$$\begin{aligned}&\int_{-\delta_2(\xi)}^{0} x^T(\nu_1(\xi-\tau_2+\theta)-\tau_1)B_1^T \cdot \\ &\qquad B_1x(\nu_1(\xi-\tau_2+\theta)-\tau_1)d\theta \\ &\le \int_{\xi-\tau_2-\varepsilon_2-\varepsilon_1-\tau_1}^{\xi-\tau_2-\tau_1} \frac{1}{1-\rho_1}x^T(\mu)B_1^TB_1x(\mu)d\mu\end{aligned} \tag{52}$$

Therefore,

$$\int_0^t u_4^T(\xi)u_4(\xi)d\xi \le \frac{(\varepsilon_1+\varepsilon_2)\varepsilon_2}{1-\rho_1}||B_1||^2 \int_0^t x^T(\mu)x(\mu)d\mu$$

Similarly, we can show

$$\int_0^t u_2^T(\xi)u_2(\xi)d\xi \le \frac{(\varepsilon_1+\varepsilon_2)\varepsilon_1}{1-\rho_2}||B_2||^2 \int_0^t x^T(\mu)x(\mu)d\mu$$

With a simpler procedure, we can also show

$$\int_0^t u_1^T(\xi)u_1(\xi)d\xi \le \varepsilon_1^2||A||^2 \int_0^t x^T(\mu)x(\mu)d\mu$$

$$\int_0^t u_3^T(\xi)u_3(\xi)d\xi \le \varepsilon_2^2||A||^2 \int_0^t x^T(\mu)x(\mu)d\mu$$

With the above discussion, we can write the system described by (47) and (43)-(46) as

$$\begin{aligned}\dot{x}(t) &= Ax(t) - B_1x(t-\tau_1) - B_2x(t-\tau_2) + \hat{B}u \\ y_i(t) &= c_ix(t), \qquad i = 1,2,3,4\end{aligned} \tag{54}$$

where

$$\begin{aligned}u(t) &= [u_1^T(t)\ u_2^T(t)\ u_3^T(t)\ u_4^T(t)]^T \\ \hat{B} &= [B_1\ B_1\ B_2\ B_2]\end{aligned}$$

and

$$c_1 = \varepsilon_1||A||, \;\; c_2 = \sqrt{\frac{(\varepsilon_1+\varepsilon_2)\varepsilon_1}{1-\rho_2}}||B_2||, \tag{55}$$

$$c_3 = \varepsilon_3||A||, \;\; c_4 = \sqrt{\frac{(\varepsilon_1+\varepsilon_2)\varepsilon_2}{1-\rho_1}}||B_1|| \tag{56}$$

with feedback $u_i(t) = \Delta_iy_i(t), \quad 1 \le i \le 4.$
With the definition of u_i and c_i, it can be easily shown that the gains of the dynamic operator Δ_i is bounded by 1.

Theorem 3 *The closed loop system is uniformally asymptotically stable for any time-varying delay uncertainty $\delta_i(t)$, $i = 1,2,3,4$, satisfiying (40), if there exist scalars α_i, $i = 1,2,3,4$ such that*

$$||\Lambda H(j\omega)\Lambda^{-1}||_\infty < \frac{1}{\epsilon_{max}}$$

where Λ= ***diag*** $(\alpha_1I_n, .., \alpha_4I_n)$, *and*

$$H(s) = \begin{bmatrix} c_1I_n \\ c_2I_n \\ c_3I_n \\ c_4I_n \end{bmatrix} (sI - A + B_1\mathrm{e}^{-\tau_1s} + B_2\mathrm{e}^{-\tau_2s})^{-1}\hat{B} \tag{58}$$

Proof: Use the small gain theorem, as discussed in Chapter 8 of [7].

4 Concluding Remarks

In this chapter, we have been interested in the closed-loop stability of some simple bilateral teleoperation scheme in the hypothesis of the existence of some communication delays. A frequency-domain approach was used to perform the stability analysis in terms of delays. The main advantage of the derived method lies in its simplicity.

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